EFFECT OF DIFFUSION AND NOISE WITH SERIES SOLUTION OF COMMENSAL- HOST SPECIES MODEL

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Abstract: *The paper propose to explore a convergent series solution in an ecological commensalism. Homotopy perturbation method is used as an efficient tool for calculating series solution. We assumed that both prey and predator species harvested at a constant rate. Commensal species drawing assets from the host species, in spite of the restriction of its natural assets superfluities. This model is categorised by a pair of non-linear attached partial differential equations. The role of dispersion on stability about interior equilibrium point and perseverance criteria of the system is studied. Finally we have examined the steadiness and dynamical behaviour of the system with a computer simulation.*

Keywords: **Commensalism, Positive Equilibrium, stability, diffusion, random noise. Homotopy Analysis.**

1. Introduction

Mathematical Modelling plays most significant role in Ecological system. In Ecology food manacles are modelled by nonlinear differential and difference equations which estimate the species behaviour relation with their habitat. Based on the previous studies [1-3] there are two species in the Ecosystem with their interaction like commensalism, Ammenslism and Mutualism were investigated. In addition to this work we study the effect of random noise and dispersion with the movement of the species with vertical direction based on the previous studies [4-6]. [7] studied the series solution of the nonlinear differential equations with two species in ecological system by using Homotopy Petirbation technique. Homotopy Perturbation Method (HPM) developed by Later Liao [8-9] in 1992. In the modern studies, the HPM methodology with numerous logic concepts and its solicitations were thoroughly developed.

The study of this paper is zealous to the analytical analysis of commensalism between two species. Here commensalism means two species living in ecosystem with same habitat one species (N_1) derives benefit from the other species (N_2).The example of commensalism is Remora-Shark interaction, where the Remora is commensal which affix itself on the host Sharks. Remoras drive on Sharks to the rich- nutrient water-regions, but neither harm nor benefit to the host by the riders. Lakshmi Narayan K et.al [10] considered a prey-predator model with shelter for prey and alternate sustenance for the predator, harvesting of both species.

The consequence of environmental fluctuations on the Commensal-Host marine ecosystem work is motivated by [11-21]. The paper is prepared as follows: In Section 2, the mathematical-model of commensalism system. The balanced states and stability (both local & global) have remained in section 3. Section 4 consists of diffusion analysis. The stability under Noise has been studied in section 5. Series solutions of given ecological model using Homotopy perturbation method in section 6**.** The section 8 contains numerical simulations and graphical representation.

2. Mathematical Formulation

Here we considered an ecological system where Commensal and Host are living together. It is assumed both species harvested at a constant rate. The populations are subject to dispersal. Let us consider the model equations for a two species multi-interactive ecosystem are given by the following system of non-linear ordinary differential equations.

(i) The equation for the growth rate of Commensal species (N_1):

$$\frac{dN_1}{dt} = a_{11}\left[k_1 N_1 - N_1^2 + c_1 N_1 N_2 - H_1\right] \tag{2.1}$$

(ii) The equation for the growth rate of host species (N_2):

$$\frac{dN_2}{dt} = a_{22}\left[k_2 N_2 - N_2^2 - H_2\right] \tag{2.2}$$

with initial conditions $N_1(0) > 0, N_2(0) > 0$

2.1. Nomenclature of the given Ecological Model:

$N_1(t)$: The population rate of the species S_1 at time t : $N_2(t)$:The population rate of the species S_2 at time t ; a_i : The natural growth rate of S_i, i = 1, 2. ; a_{ii} : The rate of decrease of Si; due to its own insufficient resources ,i=1,2. ; a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient. ; $H_1(t)$: The replenishment or renewal of S_1 per unit time $H_2(t)$: The replenishment or renewal of S_2 per unit time ; $a_{11}H_1$: the rate of harvest of the Commensal. ; $a_{22}H_2$: the rate of harvest of the Host. The state variables are assumed to be non-negative constants.

3. Analysis Of The Stability Model (2.1)-(2.2)

3.1. Existence of steady states:

The system of equations (2.1)-(2.2) without diffusive terms have the below mentioned steady states. (i)The trivial steady state $E_0(0,0)$ always exists. (ii) The axial steady state $E_1(\bar{x},0)$ exists. (iii)The axial steady state $E_2(0, y^{\phi})$ exists.(iv)The interior steady state $E_3(x^*, y^*)$ exists, where

$$\bar{x} = \frac{k_1 \pm \sqrt{k_1^2 - 4h_1}}{2}; \qquad y^{\phi} = \frac{k_2 \pm \sqrt{k_2^2 - 4h_2}}{2}.$$

For, positivity of $\bar{x}$, y^{ϕ} , we must have $h_1 > 0$, $h_2 > 0$, $k_1^2 > 4h_1$, and $k_2^2 > 4h_2$.

Again, $y^* = \dfrac{k_2 \pm \sqrt{k_2^2 - 4h_2}}{2}$.

And let $y^* = m_r$, then $x^* = \dfrac{(k_1 + c_1 m_r) \pm \sqrt{(k_1 + c_1 m_r)^2 - 4h_1}}{2}$.

For positivity of x^*, y^* we must have, $k_1 + c_1 m_r > 2\sqrt{h_1}$. The interior equilibrium is the intersection of two isoclines $x'(t) = y'(t) = 0$ in the positive quadrat of the $xy-$ plane.

3.2. Local Stability:

The local stability of positive equilibrium point can be studied by calculating the resultant variational matrix. To investigate the nature of local steadiness of the interior equilibrium $E_3\left(x^*, y^*\right)$, we work out the variational matrix about

$$J = \begin{bmatrix} a_{11}(k_1 - 2x^* + c_1 y^*) & a_{11}c_1 x^* \\ 0 & a_{22}(k_2 - 2y^*) \end{bmatrix} \qquad J(E_0) = \begin{bmatrix} a_{11}k_1 & 0 \\ 0 & a_{22}k_2 \end{bmatrix}$$

Characteristic equation of J at E_0 is $\begin{vmatrix} a_{11}k_1-\lambda & 0 \\ 0 & a_{22}k_2-\lambda \end{vmatrix}=0$

$(a_{11}k_1-\lambda)(a_{22}k_2-\lambda)=0$ we get $\lambda_1=a_{11}k_1;$ $\lambda_2=a_{22}k_2$

Since all Eigen values are positive the system is unstable at E_0

$$J(E_1)=\begin{bmatrix} a_{11}(k_1-2x^*) & a_{11}c_1x^* \\ 0 & a_{22}k_2 \end{bmatrix}$$

Characteristic equation of J at E_0 is $\begin{vmatrix} a_{11}(k_1-2x^*)-\lambda & a_{11}c_1x^* \\ 0 & a_{22}k_2-\lambda \end{vmatrix}=0$

$\left(a_{11}(k_1-2x^*)-\lambda\right)\left(a_{22}k_2-\lambda\right)=0$ weget $\lambda_1=a_{11}(k_1-2x^*);\lambda_2=a_{22}k_2$

Since all the parameters are positive, λ_2 is always positive

For λ_1 to be negative, it is required to have

$$k_1-2x^*<0\,,k_1<2x^*,x^*<\frac{k_1}{2}$$

$$J(E_2)=\begin{bmatrix} a_{11}(k_1+c_1y^*) & 0 \\ 0 & a_{22}(k_2-2y^*) \end{bmatrix}$$

Characteristic equation of J at E_0 is $\begin{vmatrix} a_{11}(k_1+c_1y^*)-\lambda & 0 \\ 0 & a_{22}(k_2-2y^*)-\lambda \end{vmatrix}=0$

$$(a_{11}(k_1+c_1y^*)-\lambda)(a_{22}(k_2-2y^*)-\lambda)=0$$

$$\lambda_1=a_{11}(k_1+c_1y^*);\lambda_2=a_{22}(k_2-2y^*)$$

When $\lambda_1>0,\lambda_2<0$ then the system is stable at E_2

Since all the parameters are positive, λ_1 is always positive

For $k_2-2y^*<0\Rightarrow k_2<2y^*\Rightarrow y^*>\frac{k_2}{2}$

Therefore the given system is stable at E_2 provided $y^*>\frac{k_2}{2}$

$$J(E_3)=\begin{bmatrix} a_{11}(k_1-2x^*+c_1y^*) & a_{11}c_1x^* \\ 0 & a_{22}(k_2-2y^*) \end{bmatrix}$$

Characteristic equation of J at E_0 is $\begin{vmatrix} a_{11}(k_1-2x^*+c_1y^*)-\lambda & a_{11}c_1x^* \\ 0 & a_{22}(k_2-2y^*)-\lambda \end{vmatrix}=0$

$$(a_{11}(k_1-2x^*+c_1y^*)-\lambda)(a_{22}(k_2-2y^*)-\lambda)=0$$

$$\lambda_1=a_{11}(k_1-2x^*+c_1y^*);\lambda_2=a_{22}(k_2-2y^*)$$

When the $\lambda_1>0 and \lambda_2<0$ then the system is

$$k_2-2x^*+c_1y^*>0\ ,\ k_1<2x^*\ ,\ x^*>\frac{k_1}{2}\ \text{and}\ \ k_2-2y^*>0\,,k_2<2y^*,y^*>\frac{k_2}{2}$$

The given system is stable at E_3 when $x^*>\frac{k_1}{2}$ and $y^*>\frac{k_2}{2}$

3.3. Global Stability:

In this fragment, we extant the essential results on the global stability of non–negative equilibrium. Consider the Lyapunov function

$$V(x,y)=\left(x-x^*-x^*\log(\frac{x}{x^*})\right)+l_1\left(y-y^*-y^*\log(\frac{y}{y^*})\right)$$

After differentiating above equation with respect to‘t’ we get

$$\frac{dV}{dt}=\left(\frac{x-x^*}{x}\right)\frac{dx}{dt}+l_1\left(\frac{y-y^*}{y}\right)\frac{dy}{dt}$$

$$=\left(\frac{x-x^*}{x}\right)\left(a_{11}[k_1x-x^2+c_1xy-h_1]\right)+l_1\left(\frac{y-y^*}{y}\right)(a_{22}(k_2y-y^2-h_2))$$

$$=\left(\frac{x-x^*}{x}\right)\left(a_{11}k_1x-a_{11}x^2+a_{11}c_1xy-a_{11}h_1\right)+l_1\left(\frac{y-y^*}{y}\right)\left(a_{22}k_2y-a_{22}y-a_{22}h_2\right)$$

$$=\left(\frac{x-x^*}{x}\right)\left(a_{11}k_1x-a_{11}x+a_{11}c_1xy-(a_{11}k_1x^*-a_{11}x^{*2}+a_{11}c_1x^*y^*)\right)$$

$$+l_1\left(\frac{y-y^*}{y}\right)\left(a_{22}k_2y-a_{22}y-(a_{22}k_2y^*-a_{22}y^{*2})\right)$$

$$=\left(\frac{x-x^*}{x}\right)\left(a_{11}k_1(x-x^*)-a_{11}(x^2-x^{*2})+a_{11}c_1(xy-x^*y^*)\right)$$

$$+l_1\left(\frac{y-y^*}{y}\right)\left(a_{22}k_2(y-y^*)-a_{22}(y-y^{*2})\right)$$

$$=\left(\frac{x-x^*}{x}\right)\left(a_{11}k_1(x-x^*)-a_{11}(x^2-x^{*2})+a_{11}c_1(xy-x^*y^*)\right)$$

$$+l_1\left(\frac{y-y^*}{y}\right)\left(a_{22}k_2(y-y^*)-a_{22}(y-y^{*2})\right)$$

$$=\left(\frac{a_{11}k_1(x-x^*)^2}{x}-\frac{a_{11}(x-x^*)^3(x+x^*)}{x}+\frac{a_{11}c_1(x-x^*)(x(y-y^*)+y^*(x-x^*))}{x}\right)$$

$$+l_1\left(\frac{a_{22}k_2(y-y^*)^2}{y}-a_{22}\frac{(y-y^*)^2(y+y^*)}{y}\right)$$

$$\frac{dV}{dt} \leq 0$$

Clearly, $V'(t) < 0$, hence the non-diffusive system (2.1)-(2.2) is globally asymptotically stable by Lyapunov Theorem

4. Diffusive Stability

In Ecosystem, we have considered commensalism interaction (Barnacle- Whale) system with constant harvesting rates. The diffusive equation system is constituted as

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - a_{11} N_1^{\,2} - a_{12} N_1 N_2 - a_{11} H_1 + D_1 \frac{\partial^2 N_1}{\partial u^2} \qquad (4.1)$$

$$\frac{\partial N_2}{\partial t} = a_2 N_2 - a_{22} N_2^{\,2} - a_{22} H_2 + D_2 \frac{\partial^2 N_2}{\partial u^2} \qquad (4.2)$$

where D_1, D_2 represents the constant diffusion coefficients of N_1&N_2 respectively

The model system (4.1)-(4.2) is in homogeneous as the reaction diffusion system. For such introduction of the diffusion term of the populations, it has become a spatio-temporal dynamical system. We consider the following conditions of the populations. $N_1(v,t),\ N_2(v,t)$ andin $0 \leq v \leq L, L > 0$ as below

$$\frac{\partial N_1(0,t)}{\partial t} = \frac{\partial N_1(L,t)}{\partial t} = \frac{\partial N_2(0,t)}{\partial t} = \frac{\partial N_2(L,t)}{\partial t} = 0$$

The zero isoclines of model equations (4.1)-(4.2) also give the steady state which are same as we have obtained for homogeneous system. Now we linearise the system (4.1)-(4.2) by putting $N_1 = X + x^*$ & $N_2 = Y + y^*$

and we obtain

$$\frac{\partial X}{\partial t} = a_{11} X x^* + a_{11} c_1 Y x^* + D_1 \frac{\partial^2 X}{\partial u^2} \qquad (4.3)$$

$$\frac{\partial Y}{\partial t} = a_{22} y^* Y + D_2 \frac{\partial^2 Y}{\partial u^2} \qquad (4.4)$$

by assuming $x = x^* + X$ and $y = y^* + Y$. Let the solution of the system (4.2)-(4.3) be the form $X(\mathrm{u},\mathrm{t}) = \alpha_1 e^{\lambda t} e^{iku}$ & $Y(u,t) = \alpha_2 e^{\lambda t} e^{iku}$; where α_1 & α_2 are amplitudes and 'k' is the wave number of the solution. X , Y are propagations of populations. Corresponding to the diffusive system (4.2)-(4.3),

the characteristic equation is $\mu^2 - A\mu + B = 0$ (4.5)

where $A = a_{11}x^* + a_{22}y^* - k^2(D_x + D_y)$; $B = a_{11}a_{22}x^* y^* + a_{11}D_y x^* k^2 - a_{22}D_x y^* k^2 + k^4 D_x D_y$. Now, our highest purpose is to originate the criteria for dispersal instability of model-system (2.1)-(2.2), for this, let us rewrite B as $G(k^2)$ where

$$G(k^2) = D_x D_y \left(k^2\right)^2 - \left(a_{11} D_y x^* + a_{22} D_x y^*\right) k^2 + a_{11} a_{22} x^* y^*.$$

The system (2.1)-(2.2) is unstable, if one of the above roots of the equation (4.5) is positive. An essential condition for a root to be positive is that

$$-k^2 (D_x + D_y) + \left(a_{11} x^* + a_{22} y^*\right) > 0,$$

which implies that $k^2 < \dfrac{a_{11}x^* + a_{22}y^*}{D_x + D_y}$ (4.6)

Since the wave number k is real number, then the above statement is achievable, if $a_{11}x^* + a_{22}y^* > 0$.

The satisfactory situation for positivity of one of the roots of the equation (4.5) is $G(k^2) < 0$. Since '$G(k^2)$' is an appearance in k^2 where 'k' the wave-number, non-zero positive quantity, the minimum of '$G(k^2)$' occurs. Let $k^2_{\min}$ be the corresponding value of k^2 for minimum value of $G(k^2)$.

Then $$k^2_{\min} = \frac{a_{11}x^* D_y + a_{22}y^* D_x}{2D_x D_y} > 0.$$

The corresponding minimum value of $G(k^2)$ is

$$G(k^2_{\min}) = -\frac{\left(a_{11}x^*\right)^2 D_y}{4D_x} + a_{11}a_{22}x^*$$

So, the sufficient condition reduces to $\Delta > \dfrac{4a_{22}}{a_{11}x^*}$ (4.6)

where $\Delta = \dfrac{D_y}{D_x}$. Thus the dispersion of the commensal-host populaces drive the environmental system into unbalanced fluctuation when (4.5) and (4.6) are satisfied.

5. Stochastic Analysis

The classical equations for a two species ecosystem with white noise are given by the subsequent system of non-linear ordinary differential equations.

$$\frac{\mathrm{dN_1}}{\mathrm{dt}} = a_1 N_1 - a_{11}N_1^2 - a_{12}N_1 N_2 - a_{11}H_1 + \Psi_1\Omega_1(t) \tag{5.1}$$

$$\frac{\mathrm{dN_2}}{\mathrm{dt}} = a_2 N_2 - a_{22}N_2^2 - a_{22}H_2 + \Psi_2\Omega_2(t) \tag{5.2}$$

Let $\Psi_1 \,\&\, \Psi_2$ are real constants, $\Omega(t) = \left[\Omega_1(t), \Omega_2(t)\right]$ is a 2D Gaussian-white- noise process satisfying $E\left[\Omega_i(t)\right] = 0 \quad i = 1,2$

$$N_1(t) = u_1(t) + s^* \,\&\, N_2(t) = u_2(t) + p^*$$

$$\frac{\mathrm{dN_1}}{\mathrm{dt}} = \frac{\mathrm{du_1}}{\mathrm{dt}} \quad \& \quad \frac{\mathrm{dN_2}}{\mathrm{dt}} = \frac{\mathrm{du_2}}{\mathrm{dt}}$$

From equation (5.1)

$$\frac{\mathrm{du_1}}{\mathrm{dt}} = a_1(u_1 + S^*) - a_{11}(u_1 + S^*)^2 + a_{12}(u_1 + S^*)(u_2 + P^*) - a_{11}H_1 + \Psi_1\Omega_1(t) \tag{5.3}$$

$$\frac{\mathrm{du_2}}{\mathrm{dt}} = a_2(u_2 + P^*) - a_{22}(u_1 + P^*)^2 - a_{22}H_2 + \Psi_2\Omega_2(t) \tag{5.4}$$

The like part of (5.3)&(5.4) is given by

$$\frac{\mathrm{du_1}}{\mathrm{dt}} = A_{11}u_1 S^* + a_{12}u_2 S^* + \psi_1\Omega_1(t) \tag{5.5}$$

$$\frac{du_2}{dt} = A_{22}u_2P^* + \psi_2\Omega_2(t) \qquad (5.6)$$

Taking Fourier technique of (5.5) & (5.6),we get

Eq (5.5) $\Rightarrow \psi_1(\omega)\,\overline{\Omega_1} = -a_{12}S^*\overline{u_2}(\omega) + \left(i\omega - A_{11}S^*\right)\overline{u_1}(\omega)$ (5.7)

Eq (5.6) $\Rightarrow \Psi_2\overline{\Omega_2}(\omega) = \left(i\omega - A_{22}P^*\right)\overline{u_2}(\omega)$ (5.8)

The matrix form of (5.7)&(5.8) is

$$\Rightarrow M(\omega)\overline{u(\omega)} = \Omega(\omega) \qquad (5.9)$$

Here $\mathrm{M}(\omega) = \begin{bmatrix} \mathrm{M}_{11} & \mathrm{M}_{12} \\ \mathrm{M}_{21} & \mathrm{M}_{22} \end{bmatrix}$; $\bar{u}(\omega) = \begin{bmatrix} u_1(\omega) \\ u_2(\omega) \end{bmatrix}$; & $\bar{\Omega}(\omega) = \begin{bmatrix} \psi_1\bar{\Omega}_1(\omega) \\ \psi_2\bar{\Omega}_2(\omega) \end{bmatrix}$

where $\mathrm{M}(\omega) = \begin{bmatrix} i\omega - A_{11}S^* & a_{12}S^* \\ 0 & i\omega - A_{22}P^* \end{bmatrix}$

$$\Rightarrow \left|\det M(\omega)\right| = R(\omega)^2 + I(\omega)^2$$

$$\Rightarrow \left|\det M(\omega)\right|^2 = (-\omega^2 + A_{11}A_{22}S^*P^*)^2 + (A_{11}S^* + A_{22}P^*)^2\omega^2$$

$$From\ (5.13) \Rightarrow \bar{u}(\omega) = \left[M(\omega)\right]^{-1} + \bar{\Omega}(\omega)$$

where $\left[\mathrm{M}(\omega)\right]^{-1} = k(\omega) = \dfrac{1}{\mathrm{M}(\omega)}\begin{bmatrix} \mathrm{M}_{11}{}^{CF(1,1)^T}(\omega) & \mathrm{M}_{21}{}^{CF(1,2)^T}(\omega) \\ \mathrm{M}_{12}{}^{CF(2,1)^T}(\omega) & \mathrm{M}_{22}{}^{CF(2,2)^T}(\omega) \end{bmatrix}$

$$Now\ \sigma_{u_1}{}^2 = \frac{1}{2\pi}\sum_{i=1}^{3}\int_{-\infty}^{\infty}\Psi_i\left|\frac{M_{i1}{}^{CF(1,i)^T}}{|M(\omega)|}\right|^2 d\omega, \sigma_{u_2}{}^2 = \frac{1}{2\pi}\sum_{i=1}^{3}\int_{-\infty}^{\infty}\Psi_i\left|\frac{M_{i2}{}^{CF(2,i)^T}}{|M(\omega)|}\right|^2 d\omega,$$

$$\Rightarrow \left|M_{11}{}^{CF(1,1)^T}(\omega)\right|^2 = \omega^2 + A_{22}{}^2P^{*2}; \qquad \left|M_{12}{}^{CF(2,1)^T}(\omega)\right|^2 = a_{12}{}^2S^{*2}$$

$$\left|M_{21}{}^{CF(1,2)^T}(\omega)\right|^2 = 0; \qquad \left|M_{22}{}^{CF(2,2)^T}(\omega)\right|^2 = \omega^2 + A_{11}{}^2S^{*2}$$

$Case(1):\ If\ \Psi_1 = 0$

$$\sigma_{u_1}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Psi_2 a_{12}{}^2S^{*2}}{|M(\omega)|^2}d\omega$$

$$\sigma_{u_2}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Psi_2(\omega^2 + A_{11}{}^2S^{*2})}{|M(\omega)|^2}d\omega$$

$Case(2):\ If\ \Psi_2 = 0$

$$\sigma_{u_1}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Psi_1(\omega^2 + A_{22}{}^2P^{*2})}{|M(\omega)|^2}d\omega$$

$$\sigma_{u_2}{}^2 = 0$$

Clearly the balance of populaces for smaller estimations of mean square uncertainties is pointed out by the population variances.

6. Series Solutions By Homotopy Perturbation Method

Let us take the nonlinear differential equation:

$$Au - fr = 0, \quad r \in \Omega \tag{6.1}$$

With the stationary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \quad , r \in \Gamma$$

where A is a normal differential operator, B a boundary operator, $f\ r$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω .

In general the operator A, is divided into two parts: a linear part L and a nonlinear part N. Therefore above differential equation(6.1) is expressed in the form of A

$$Lu - Nu - fr = 0 \tag{6.2}$$

With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy

$v\,r, p : \Omega \times 0,1 \rightarrow R$ which satisfies

$$H\ v, p = 1 - p(L\,v - L\,u_0) + p(A\,v - f\ r) = 0 \quad p \in 0,1 \quad r \in \Omega \tag{6.3}$$

It is nothing but

$$H\ v, p = (L\,v - L\,u_0) + pL\,u_0 + p(N\,v - f\ r) = 0 \tag{6.4}$$

Where $p \in 0,1$ is named as an embedding parameter ,and u_0 is an initial approximation of equation(6.1),which satisfies the boundary conditions

Then equations (6.3), (6.4) follow that

$H\ v,0 = (L\,v - L\,u_0) = 0$ and $H\ v,1 = (A\,v - f\ r) = 0$

Thus the changing process of p from zero to unity is just that of v, r, p from u_0, r to u, r .

According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (6.3) and 6.4) can be written as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + p^5 v_5 + ...$$

The approximate solution of equation (6.1) can be obtained as

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + v_4 + v_5 + ...$$

Basic Equations of the given ecological model is

$$\frac{dN_1}{dt} = a_{11}\left[k_1 N_1 - N_1^{\ 2} + c_1 N_1 N_2 - H_1\right] \tag{6.5}$$

$$\frac{dN_2}{dt} = a_{22}\left[k_2 N_2 - N_2^{\ 2} - H_2\right] \tag{6.6}$$

With initial conditions $N_1(0) = c_1, \& N_2(0) = c_2$

The following system can be constructed by the concept of homotopy as follows

$$v_1^{\ 1} - N_{10}^{\ \ 1} + p(N_{10}^{\ \ 1} - a_1 v_1 - a_{11} v_1^2 + a_{12} v_1 v_2 - a_{11} H_1) = 0 \tag{6.7}$$

$$v_2^{\ 1} - N_{20}^{\ \ 1} + p(N_{20}^{\ \ 1} - a_2 v_2 - a_{22} v_2^{\ 2} - a_{22} H_2) = 0 \tag{6.8}$$

The initial approximations are considered as

$$v_{1,0}(t)=N_{10}(t)=v_1(0)=c_1 \quad (6.9)$$

$$v_{2,0}(t)=N_{20}(t)=v_2(0)=c_2 \quad (6.10)$$

And

$$v_1(t)=v_{1,0}(t)+pv_{1,1}(t)+p^2v_{1,2}(t)+p^3v_{1,3}(t)+p^4v_{1,4}(t)+\dots \quad (6.11)$$

$$v_2(t)=v_{2,0}(t)+pv_{2,1}(t)+p^2v_{2,2}(t)+p^3v_{2,3}(t)+p^4v_{2,4}(t)+\dots \quad (6.12)$$

The 4-terms approximations are sufficient, we get

$$N_1(t)=\lim_{p\to 1} v_1(t)=\sum_{x=0}^{4} v_{1,x}(t)=v_{1,0}(t)+p\,v_{1,1}(t)+p^2v_{1,2}(t)+p^3v_{1,3}(t)+p^4v_{1,4}(t)$$

$$N_2(t)=\lim_{p\to 1} v_2(t)=\sum_{x=0}^{4} v_{2,x}(t)=v_{2,0}(t)+p\,v_{2,1}(t)+p^2v_{2,2}(t)+p^3v_{2,3}(t)+p^4v_{2,4}(t)$$

The convergent series solutions are obtained by HPM as

$$\begin{aligned}
N_1(t)= {} & c_1+(a_1-a_{11}c_1-a_{12}c_2)c_1t \\
& +[(a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_1+a_{11}c_1-a_{12}c_2)+(a_{11}c_1-a_{12}c_1)(a_2c_2-a_{22}c_2{}^2)]\frac{t^2}{2} \\
& +[(a_1+2a_{11}c_1+2a_{12}c_2)(a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_1+a_{11}c_1-a_{12}c_2)+(a_{11}c_1-a_{12}c_1)(a_2c_2-a_{22}c_2{}^2) \\
& +2a_{11}(a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)+a_{12}c_1(a_2+2a_{22}c_2)(a_2c_2-a_{22}c_2{}^2))]\frac{t^3}{6} \\
& +[(a_1+2c_1a_{11}+c_2a_{12})((a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_1+a_{11}c_1-a_{12}c_2)(a_1+2a_{11}c_1+2a_{12}c_2) \\
& +(a_1c_1-a_{12}c_2)(a_2c_2-a_{22}c_2{}^2))+2a_{11}((a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)+a_{12}c_1(a_2+2a_{22}c_2)(a_2c_2-a_{22}c_2{}^2)) \\
& +2a_{11}((a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)((a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_1+a_{11}c_1-a_{12}c_2)+(a_{11}c_1-a_{12}c_1)(a_2c_2-a_{22}c_2{}^2))) \\
& +a_{12}c_1((a_2c_2-a_{22}c_2{}^2)(a_2+2a_{22}c_2)+(a_2+2a_{22}c_2-a_{22})) \\
& +a_{12}((a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_2+2a_{22}c_2)(a_2c_2-a_{22}c_2{}^2)) \\
& +a_{12}(a_1c_1-a_{11}c_1{}^2+a_{12}c_1c_2)(a_1+a_{11}c_1-a_{12}c_2)+(a_{11}c_1-a_{12}c_1)(a_2c_2-a_{22}c_2{}^2)^2)]\frac{t^4}{24}
\end{aligned}$$

$$\begin{aligned}
N_2(t)= {} & c_2+(a_2c_2-a_{22}c_2{}^2)t+((a_2c_2-a_{22}c_2{}^2)(a_2+2a_{22}c_2))\frac{t^2}{2} \\
& +((a_2c_2-a_{22}\,c_2{}^2)(a_2+2a_{22}c_2)(a_2+2a_{22}c_2-a_{22}))\frac{t^3}{3} \\
& +((a_2+2a_{22}c_2)^2(a_2c_2-a_{22}c_2{}^2)((a_2+2a_{22}c_2)(a_2+2a_{22}c_2-a_{22})+6a_{22}(a_2c_2-a_{22}c_2{}^2)))\frac{t^4}{24}
\end{aligned}$$

7. Numerical Simulations

For verification of our results from the above segments, here we give some numerical reproductions using Matrix Laboratory.

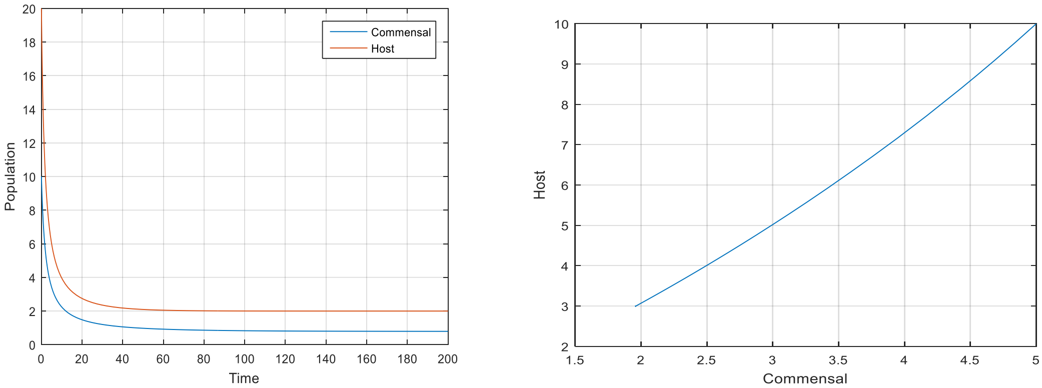

Figure (1)

Figure (1) (left) indicates the population growth against period and figure (1) (right) indicates the phase-portrait dynamics of commensal and host species for the parameters $a_1 = 3.51;\ a_{11} = 0.012;$ $a_{12} = 0.52;\ H_1 = 0.023;\ a_2 = 1.751; a_{22} = 0.52; H_2 = 30.$

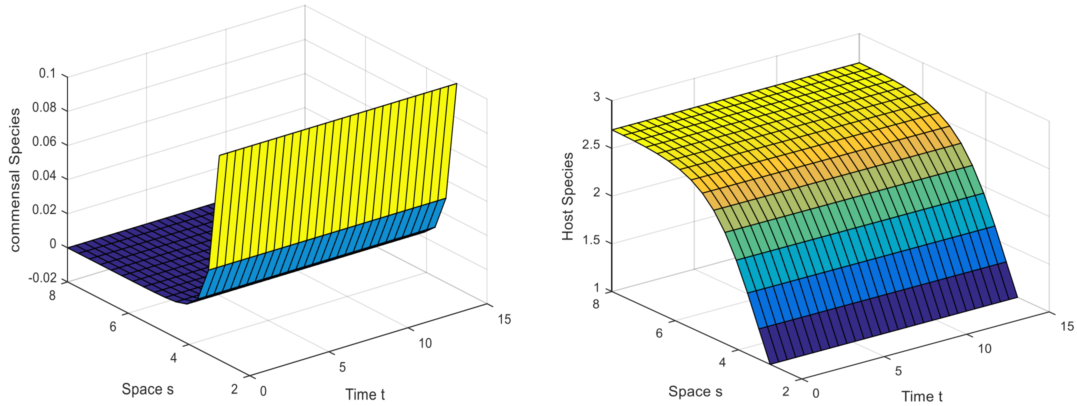

Figure (2)

Figure(2) indicates the Stable oscillations for movement of the Commensal-Host against Period and Space $a_1 = 3.51;\ a_{11} = 0.012;\ a_{12} = 0.52;\ H_1 = 0.023;\ a_2 = 1.751; a_{22} = 0.52; H_2 = 30.$ $D_1 = 5;$ $D_2 = 0.0012$

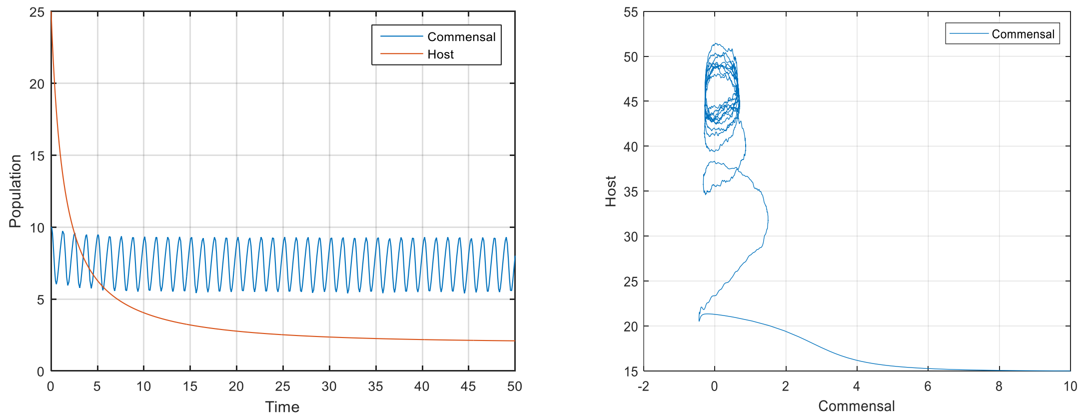

Figure (3)

Figure (3.) (left) indicates the population growth against time in the presence of random noise for commensal and host species and figure (3.) (right) shows the phase-patriot dynamics of commensal and host species for the parameters $a_1 = 3.51;\ a_{11} = 0.012;\ a_{12} = 0.52;\ H_1 = 0.023;$ $a_2 = 1.751; a_{22} = 0.52; H_2 = 30.\ \psi_1 = 5\ .\psi_2 = 10$.

8.Conclusions

An Investigation on a Mathematical Model of Significant two Species Ecosystem has been thoroughly done and the following conclusions are drawn:

(i).Identified the behaviour of the system with perturbation technique .The two stable cases are occurred in co-existence state.

(ii).Geometric interpretation is performed to illustrate the asymptotic stable.

(iii).Local stability is noticed at interior equilibrium state by Routh-Hurwitz criterion

(iv).Global Stability of the system is observed by constructing suitable Lyapunov function.

(v).The stability of the system is discussed with Diffusion and Stochastic Analysis.

(vi).The convergent series solutions of this system are derived by Homotopy Perturbation Method.

Acknowledgments

This paper was presented in ALC (Andra Loyola college) -2021 on 08-01-2021.

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